Generalized Geometric Quantum Speed Limits

D. P. Pires, M. Cianciaruso, L. C. Céleri, G. Adesso, and D. O. Soares-Pinto

Phys. Rev. X **6**, 021031 (2016)

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

BACKGROUND

QUANTUM SPEED LIMITS

- Quantum speed limits are lower bounds to the time τ that a quantum system takes to undergo a given dynamics between an initial and a target quantum state.
- Establishing general and tight quantum speed limits is crucial to assess how fast quantum technologies can ultimately be.
- Two examples of quantum speed limits for a unitary dynamics between two orthogonal pure states and generated by a time-independent Hamiltonian H are:

$$\tau \geq \frac{\pi\hbar}{2\Delta E}$$ **Mandelstam-Tamm** bound

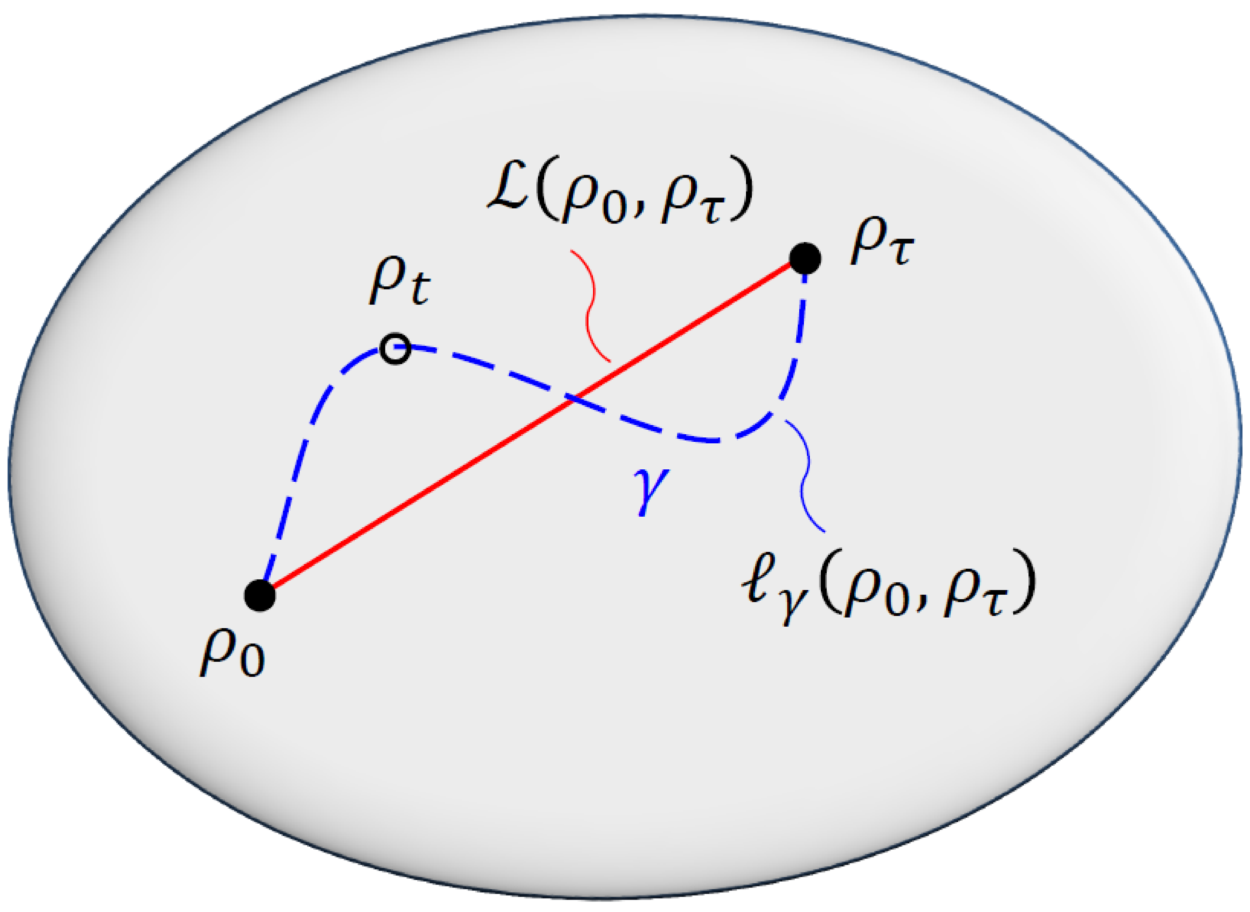
$$\tau \geq \frac{\pi\hbar}{2E}$$ **Margolus-Levitin** bound

where $(\Delta E)^2 = \langle (H - \langle H \rangle)^2 \rangle$ and $E = \langle H \rangle$.

- Another example valid for any physical dynamics γ is:

$$\mathcal{L}^{BU}(\rho_0, \rho_\tau) \leq \ell_\gamma^{BU}(\rho_0, \rho_\tau)$$

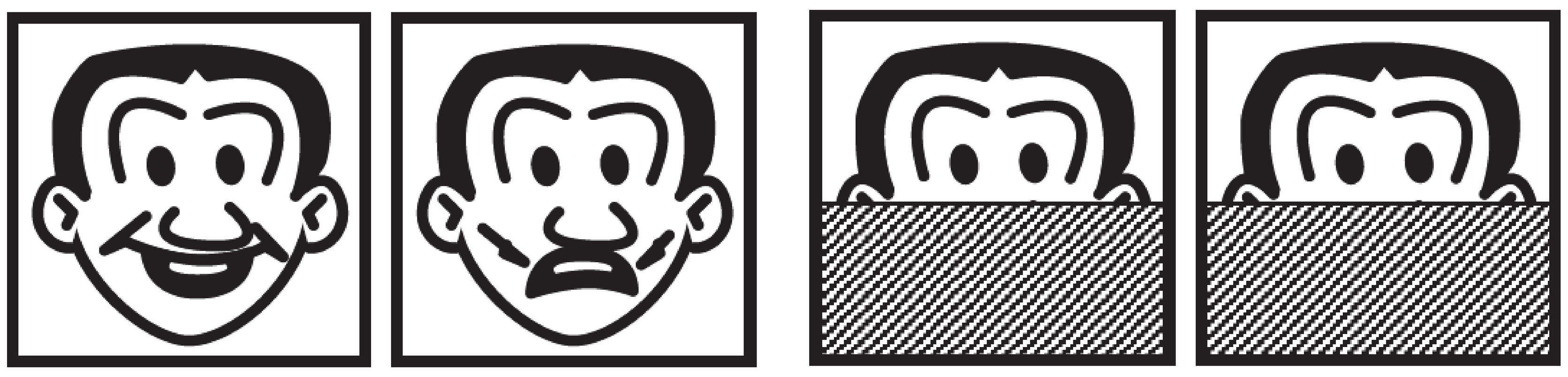
where $\mathcal{L}^{BU}$ and ℓ_γ^{BU} are, respectively, the geodesic distance and the length of the path γ corresponding to the Bures-Uhlmann metric.



QUANTUM STATE DISTINGUISHABILITY

- The set of states of a quantum system is the Riemannian manifold of density operators ρ over the system Hilbert space.
- It is natural to use any of the possible **contractive** Riemannian metrics defined on such set of states to distinguish any two of its points.
- A Riemannian metric is contractive if the corresponding geodesic distance $\mathcal{L}$ contracts under physical maps Λ:

$$\mathcal{L}(\rho, \sigma) \geq \mathcal{L}(\Lambda(\rho), \Lambda(\sigma))$$



- According to the Morozova, Čhencov, and Petz theorem, such metrics are in one-to-one correspondence with the Morozova-Čhencov functions f as follows:

$$ds^2 = \frac{1}{4}\left(\sum_j \frac{(dp_j)^2}{p_j} + 2\sum_{j<k} c^f(p_j, p_k)\left|d\rho_{jk}\right|^2\right)$$

where ds^2 is the squared infinitesimal distance between the states $\rho = \sum_j p_j |j\rangle\langle j|$ and $\rho + d\rho$, and $c^f(x,y) \equiv \frac{1}{yf(x/y)}$.

Two notable examples are:

$$c^{BU}(x,y) = \left(\frac{x+y}{2}\right)^{-1}$$ **Bures-Uhlmann** metric

$$c^{WY}(x,y) = \left(\frac{\sqrt{x}+\sqrt{y}}{2}\right)^{-2}$$ **Wigner-Yanase** metric

RESULTS

GENERALIZED GEOMETRIC QUANTUM SPEED LIMITS

- We exploit the fact that more than one privileged contractive Riemannian metric appears in quantum mechanics in order to introduce a new infinite family of quantum speed limits valid for any physical process:

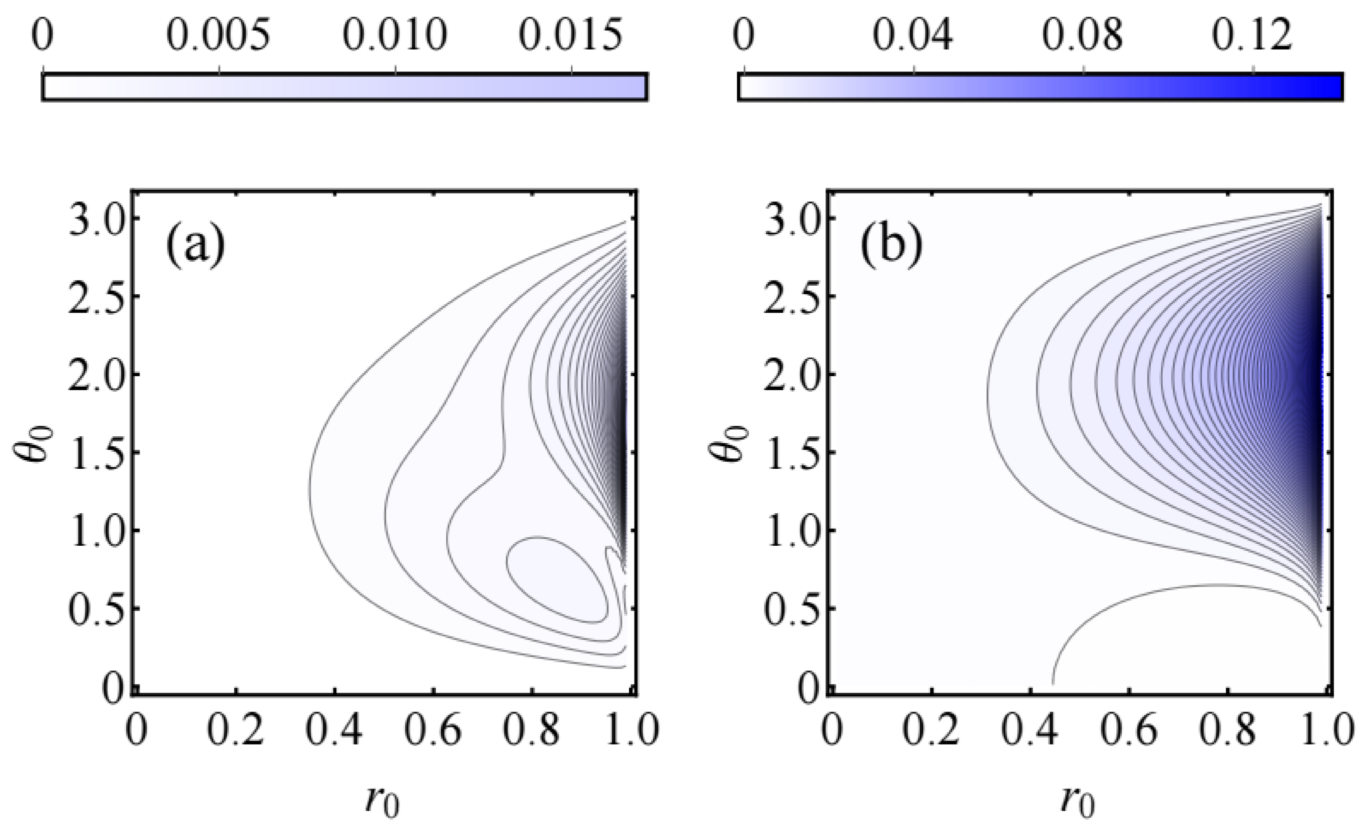
$$\mathcal{L}^f(\rho_0, \rho_\tau) \leq \ell_\gamma^f(\rho_0, \rho_\tau)$$

- The contractive Riemannian metric $\boldsymbol{g}^f$ whose geodesic is most tailored to the given dynamics γ, i.e., the one minimizing the following tightness indicator:

$$\delta_\gamma^f(\rho_0, \rho_\tau) \equiv \frac{\ell_\gamma^f(\rho_0, \rho_\tau) - \mathcal{L}^f(\rho_0, \rho_\tau)}{\mathcal{L}^f(\rho_0, \rho_\tau)}$$

provides the tightest geometric quantum speed limit.

WIGNER-YANASE CAN BEAT BURES-UHLMANN!



Contour plots of (a) δ_γ^{WY} and (b) $\delta_\gamma^{BU} - \delta_\gamma^{WY}$ when ρ_0 is a one-qubit state whose Bloch sphere representation has radial distance r_0, polar angle θ_0 and whatever azimuthal angle φ_0, while ρ_τ is its evolution at $\tau = 10$ after amplitude damping.