Information Geometry of Quantum Resources

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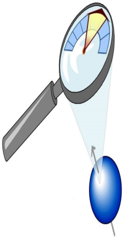
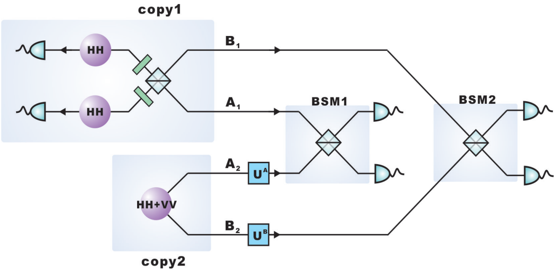




Liblice, 15 June 2016

Credits

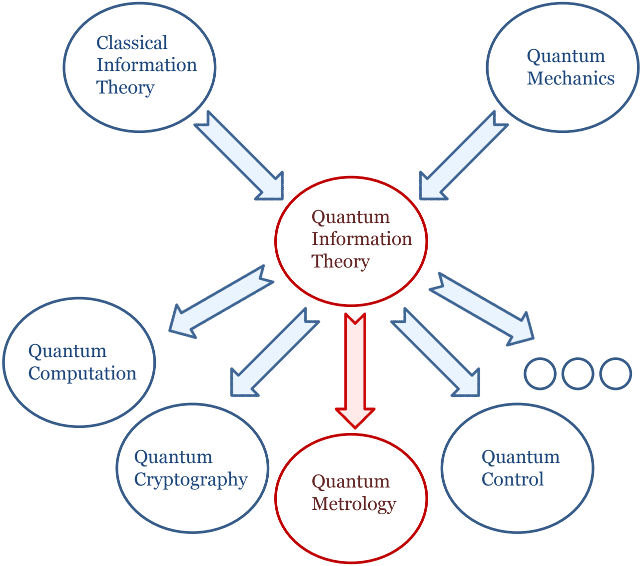
Theory

⋆ B. Yadin and V. Vedral (Oxford)

Experiment

⋆ C. Zhang, Y.-F. Huang, C.-F. Li (Hefei)

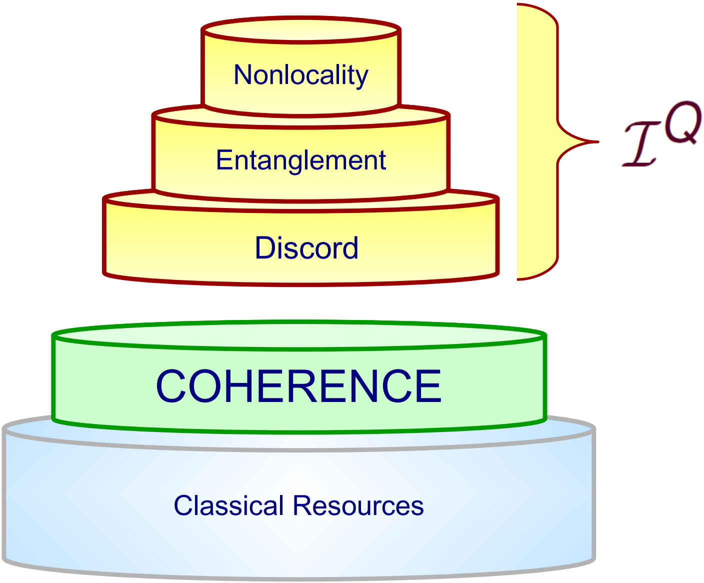
Quantum Information Theory



Resource Theory Approach

- Laboratory degrees of freedom $\Rightarrow$ Parameters
- Physical Systems and Dynamics $\Rightarrow$ Logic Resources
- Laws of Physics $\Rightarrow$ Constraints defining *free* states and operations

Hierarchy of Logic Resources



Goal: Characterizing Quantum Resources

- **?** Theoretical Quantification: $f_R(\rho)$ being monotone under free operations

- **?** Demonstration of supraclassical performance: $f_R(\rho)$ is figure of merit in a task

- **?** Experimental Detection:

  $f_R(\rho) = \langle O_{\text{exp}} \rangle, O_{\text{exp}} = O_{\text{exp}}^{\dagger}$

Coherence: Theoretical Quantification

Determining the Free operations: hard!

Candidates:

GIO $\subset$ PIO $\subset$ TIO $\subset$ SIO $\subset$ DIO $\subset$ IO

Safe Choice: $\mathrm{IO} = \{\{K_i\} : \sum_i K_i \rho K_i^\dagger = \sigma \in \mathcal{I}, \forall \rho \in \mathcal{I}\}, K_i = \sum_i c_i |i\rangle\langle g(i)|, \forall K_i \in \mathrm{IO}$

Coherence monotone under IO

$f_C(\rho) = \min_{\sigma \in \mathcal{I}} d(\rho, \sigma) = d(\rho, \tilde{\sigma})$

$S(\rho || \tilde{\sigma}_S) = \mathrm{Tr}[\rho \log \rho] - \mathrm{Tr}[\rho \log \tilde{\sigma}_S]$

$d_B(\rho, \tilde{\sigma}_B) = 2 - 2F_B(\rho, \tilde{\sigma}_B), F_B(\rho, \sigma) = \mathrm{Tr}[\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}]$

T. Baumgratz, M. Cramer, M. B. Plenio, PRL 113, 140401 (2014)

Coherence: Theoretical Quantification/2

NO operational interpretation for IO. Let's consider TIO

TIO $= \{\{K_i\} : \sum_i K_i U_\phi \rho U_\phi^\dagger K_i^\dagger = \sum_i U_\phi K_i \rho K_i^\dagger U_\phi^\dagger\}$
$K_i = \sum_{l,m:h_l - h_m = \delta} c_{lm} |h_l\rangle\langle h_m|, \forall K_i \in$ TIO, $H = \sum_l h_l |h_l\rangle\langle h_l|$

Monotone for TIO

$$\mathcal{F}(\rho_\phi, \phi) = \sum_{i,j} \frac{(\lambda_i - \lambda_j)^2}{\lambda_i + \lambda_j} |\langle i|H|j\rangle|^2$$

I. Marvian and R. Spekkens, Nature Comm. 5, 3821 (2014), D. Girolami, PRL 113, 170401 (2014), D. Girolami and B. Yadin, arXiv:1509.04131, B. Yadin and V. Vedral, Phys. Rev. A 93, 022122 (2016)

Coherence $\Rightarrow$ *Quantum Uncertainty*

- Quantum state and observable: ρ, K
- Total Uncertainty: $V(\rho, K) = \mathrm{Tr}[\rho K^2] - \mathrm{Tr}[\rho K]^2$
- Quantum Uncertainty (guess): $f([\rho, K])$
- Pure states: $V = \mathcal{I}$; Mixed states: $V \geq \mathcal{I}$

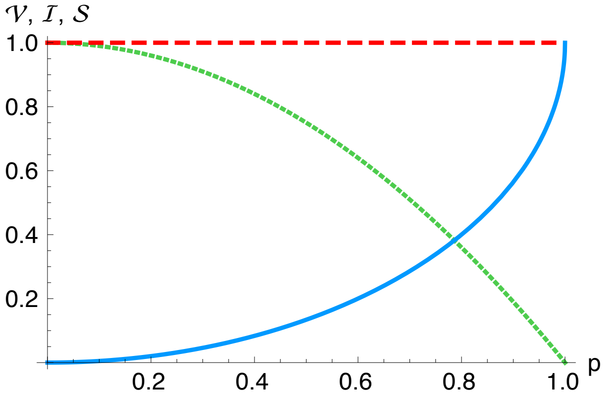
$$\mathcal{I}(\rho, K) = -\frac{1}{2}\mathrm{Tr}\left[[\sqrt{\rho}, K]^2\right]$$

Nonnegative, convex, nonincreasing under TIO channels $\Rightarrow$ *bona fide* measure of coherence

E.P. Wigner and M.M. Yanase, PNAS 49, 910 (1963), S. Luo, Phys. Rev. Lett. 91, 180403 (2003), F. Herbut, JPA 38, 2959 (2005), D. Girolami PRL 113, 170401 (2014)

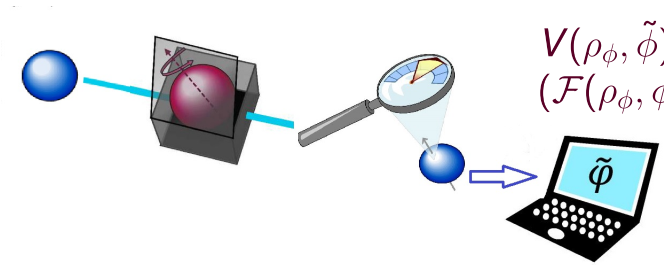
Theory: Coherence as *quantum* uncertainty

$$\rho = \frac{(1-p)}{2}\mathbb{I}_2 + p|\psi\rangle\langle\psi|, |\psi\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



Quantum Uncertainty $\Rightarrow$ Sensitivity to unitary phase shift

Given ρ, $U_\phi = e^{-i\phi H}$, estimate ϕ by measuring $U_\phi \rho U_\phi^\dagger$



$$V(\rho_\phi, \tilde{\phi}) \geq (\mathcal{F}(\rho_\phi, \phi)\nu)^{-1}$$

C. W. Helstrom, Quantum Detection and Estimation Theory (Academic Press, 1976),
S. L. Braunstein and C. M. Caves, Phys. Rev. Lett. 72, 3439 (1994)

Sensitivity $\Rightarrow$ *Coherence*

$\mathcal{F}(\rho_\phi, \phi)$ is a parent quantity of $\mathcal{I}(\rho, H)$!!!

Nonnegative, convex, nonincreasing under TIO channels $\Rightarrow$ *bona fide* measure of coherence

D. Girolami and B. Yadin, arXiv:1509.04131, B. Yadin and V. Vedral, Phys. Rev. A 93, 022122 (2016)

Efficient Detection of the Quantum Fisher Information

A very powerful result

$$f_k(\rho) = \sum_i c_i \langle O^i_{\mathrm{exp}} \rangle_{\bigotimes_{l=1}^{k} \rho_l}$$

A lower bound of the Fisher information satisfies the condition!!!

$$\mathcal{F}(\rho_\phi, \phi) \geq -1/4 \mathrm{Tr}[[\rho, K]^2]$$

EXP: Evaluation of coherence by *only two* measurements

$$-\frac{1}{4}\mathrm{Tr}\left[[\rho, K]^2\right] = -\frac{1}{\phi^2}(\mathrm{Tr}[\rho^2] - \mathrm{Tr}[\rho U_K(\phi)\rho U_K(\phi)^\dagger]) + O(\phi),$$
$$U_K(\phi) = e^{iK\phi}$$

Observable quantities

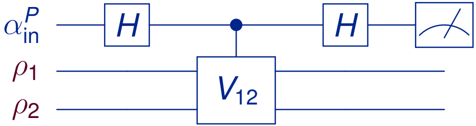
A: $\boxed{\mathrm{Tr}[\rho^2] = \mathrm{Tr}[V_{12}(\rho_1 \otimes \rho_2)]}$

B: $\boxed{\mathrm{Tr}[\rho U_K(\phi)\rho U_K(\phi)^\dagger] = \mathrm{Tr}[V_{12}(\rho_1 \otimes U_K(\phi)\rho_2 U_K(\phi)^\dagger)]}$

EXP: Schemes

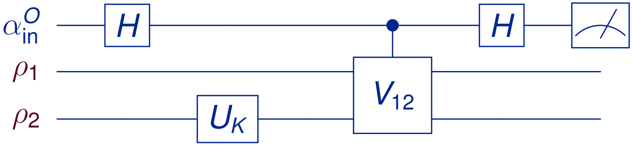
A

$$\langle \sigma_z \rangle_{\alpha^P_{\text{out}}} = \text{Tr}[\rho_1 \rho_2]$$

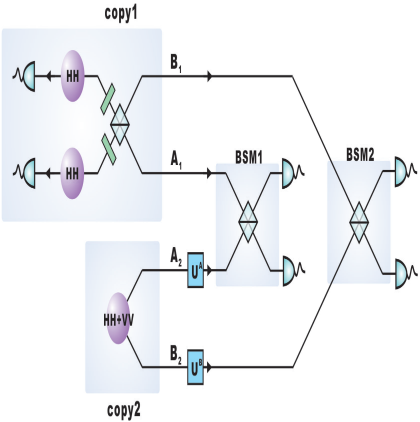


B

$$\langle \sigma_z \rangle_{\alpha^O_{\text{out}}} = \text{Tr}[\rho_1 U_K(\phi) \rho_2 U_K(\phi)^\dagger]$$



Proof of concept experiment by 4-photon source



- ✓ Standard testbed for quantum information experiments
- ✓ High fidelity for quantum operations
- ✓ Anyway, results are setup-independent

Summary

- ✓ TH: Full-fledged theoretical measure of quantum coherence for states of finite dimensional systems

- ✓ EXP: Practical schemes to quantify coherence without tomography

Thank you!

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