Higher Order Analysis of Bayesian Cross Validation in Regular Asymptotic Theory

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In Bayesian estimation, optimization of the hyperparameter is one of the most important problems. Since the effect of the hyperparameter choice to the generalization loss can not be derived by the lower order asymptotic theory, we need the higher order statistics using tensor analysis. In this talk, we introduce the higher order analysis of the Bayesian cross validation loss and prove the following results. Firstly, the cross validation loss and the average generalization loss are minimized by the common hyperparameter asymptotically. Secondly, such a hyperparameter does not minimizes the random generalization loss even asymptotically. And lastly, the information criterion WAIC has the same higher order asymptotic behavior as the Bayesian cross validation loss. We also show that minimizing the cross validation loss is different from maximizing the marginal likelihood.