Generalized Geometric Quantum Speed Limits

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The attempt to gain a theoretical understanding of the concept of time in quantum mechanics has triggered significant progress towards the search for faster and more efficient quantum technologies. One of such advances consists in the interpretation of the time-energy uncertainty relations as lower bounds for the minimal evolution time between two distinguishable states of a quantum system, also known as quantum speed limits (QSLs)[1].

Distinguishing between two states of a system being described by a probabilistic model stands as the paradigmatic task of information theory. Information geometry, in particular, applies methods of differential geometry in order to achieve this task [2]. The set of states of both classical and quantum systems is indeed a Riemannian manifold, that is the set of probability distributions over the system phase space and the set of density operators over the system Hilbert space, respectively. Therefore it seems natural to use any of the possible Riemannian metrics defined on such sets of states in order to distinguish any two of its points. However, it is also natural to assume that for a Riemannian metric to be bona fide in quantifying the distinguishability between two states, it must be contractive under the physical maps that represent the mathematical counterpart of noise, i.e. stochastic maps in the classical settings and completely positive trace preserving maps in the quantum one. Interestingly, Čencov's theorem states that the Fisher information metric is the only Riemannian metric on the set of probability distributions that is contractive under stochastic maps, thus leaving us with only one choice of bona fide Riemannian geometric measure of distinguishability within the classical setting. On the contrary, it turns out that the quantum Fisher information metric, also known as Bures-Uhlmann metric, is not the only contractive Riemannian metric on the set of density operators, but rather there exists an infinite family of such metrics, as characterized by the Morozova, Čencov and Petz theorem.

We construct a new fundamental family of geometric QSLs which is in one to one correspondence with the family of contractive Riemannian metrics characterized by the Morozova, Čencov and Petz theorem. We demonstrate how such non uniqueness of a bona fide measure of distinguishability defined on the quantum state space affects the QSLs and can be exploited in order to look for tighter bounds. Our approach is general enough to provide a unified picture, encompassing both unitary and nonunitary dynamics. We show explicit instances of QSLs which make use of some particular contractive Riemannian metric such as the Wigner-Yanase skew information, and can be provably tighter than the corresponding QSLs obtained with the conventional quantum Fisher information, thus highlighting the power of our general approach to reach beyond the state of the art.

Keywords: Quantum Speed Limits, Information Geometry.

References

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