Volume of the space of qubit channels and the distribution of some scalar quantities on it

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In quantum information theory, a qubit channel is the simplest quantum analogue of a probability transition matrix known from Kolmogorovian probability theory. The space of qubit channels can be identified with a convex submanifold of $\mathbb{R}^{12}$ via Choi representation [2]. To each qubit channel a classical channel can be associated which is called the underlying classical channel.

Our main goal is to investigate the distribution of scalar quantities, which are interesting in information geometrical point of view, on the space of qubit channels.

Our approach based on the positivity criterion for self-adjoint matrices by means of the left upper submatrices. This method was previously succesfully applied by A. Andai to compute the volume of density matrices [1].

The volume of the space of qubit channels with respect to the canonical Eucledian measure is computed, and explicit formulas are presented for the distribution of the volume over classical channels. We have constructed an efficient algorithm for generating uniformly distributed points in the space of qubit channels which enables us to investigate numerically the distribution of scalar quantities on the whole space or over a fixed classical channel. Distribution of trace-distance contraction coefficient (η^{Tr}) was investigated numerically by Monte-Carlo simulations. We computed the distribution of the Hilbert-Schmidt distance between the indentity and its image under the action of a qubit channel.

The range of possible values of η^{Tr} over an arbitrary fixed classical channel was determined explicitely and the mode of η^{Tr} was calculated numerically. We have found that the distribution of trace-distance contraction coefficient shows drammaticaly different behaviour over real and complex unital qubit channels.

Keywords: Qubit channel, trace-distance contraction coefficient, Choi matrix, volume, Monte-Carlo methods.

References

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[2] Man Duen Choi (1975), Completely positive maps on complex matrices, *Linear Algebra and Applications*, 10:285–290.